7.1 Thought-Traps for the Unwary (p. 161)

A Proof that 1 = 2

A FAMILIAR ARITHMETICAL PARADOX is a "proof" that 1 = 2. Here is one version of it:[1]

1. Whatever x is, it's obvious that:	$x = x$
2. Square both sides:	$x^2 = x^2$
3. Subtract x^2 from both sides:	$x^2 - x^2 = x^2 - x^2$
4. Factor both sides:	$x(x - x) = (x + x)(x - x)$
5. Divide both sides by (x - x):	$x = (x + x)$
6. Divide both sides by x:	$1 = (1 + 1)$
7. So:	$1 = 2$

Where is the mistake?

The mistake here is in step 5, where we divide by (x - x), which is 0. Division by 0 isn't allowed.

Quine uses a version of this "proof" and the Barber Paradox to distinguish between kinds of paradox. The "proof" that 1 = 2 uses incorrect reasoning that purports to show that something that is really false is true. Quine calls examples of this sort of reasoning "falsidical" paradoxes. A "veridical" paradox, by contrast, uses correct reasoning. When it comes up with an apparently false conclusion, we must conclude either that the conclusion is true after all, or that one of the assumptions made in the course of the reasoning was incorrect. In the Barber Paradox, the incorrect assumption is that there exists—or even could exist—such a town with such a barber.

FOR FURTHER READING: William Poundstone, in *The Labyrinths of Reason* (New York: Doubleday, 1988), gives another version of the "proof." This book contains interesting discussions of paradoxes.

---

1 Thanks to Ted Cohen, who suggested this version of the paradox to me in a letter.