

## 5.5 Paradoxes of Self-Reference (p. 126)

### *Lawyers Are Like That*

THIS TRADITIONAL PUZZLE CONCERNS the ancient Greek Protagoras (c. 480–411 BCE—the same person who turns up in one of Plato’s dialogues). Protagoras taught law, and he had this generous contract with Euathlus, one of his students: Euathlus is to pay Protagoras for his education if he wins his first case; if he doesn’t win, he doesn’t have to pay.

Having just graduated, Euathlus sues Protagoras for free tuition. Euathlus gloatingly tells Protagoras: If I win this case, I get free tuition, since that’s what I’m suing for. This is my first case, so if I lose, I don’t have to pay tuition—that’s what the contract says. So either way I won’t have to pay.

Protagoras replies: If you win, you’ll have to pay me, because that’s what our contract says. If you lose your suit for free tuition, the court will order you to pay tuition, so you’ll have to pay me. Either way, you’ll have to pay.

**A QUESTION TO THINK ABOUT:** Do you think either of the two is right? Suppose you were the judge hearing this case in court. What would you do?

### *Defining Logical Validity*

LOGICIANS DEFINE ‘VALIDITY’ THIS way: It’s the property of some arguments such that, *were* the premises true, the conclusion *would be* true. So this can be a property of arguments with all true premises, or with some or all false premises.

Following are two valid arguments (with premises marked with ‘P’ and conclusions marked with ‘C’):

#### **ARGUMENT 1**

P: All birds have wings.

P: All pigeons are birds.

C: THEREFORE all pigeons have wings.

#### **ARGUMENT 2**

P: All pigs can fly.

P: All snakes are pigs.

C: THEREFORE all snakes can fly.

Both of these arguments are valid. That means that IF the premises were true, then it would have to be the case that the conclusion was true too. Think about both. In the case of Argument 1, imagine that both premises are true (it doesn't take a great deal of imagination, because they both *are* true). Given that, it would have to be the case that the conclusion was true, right? Right. And in the case of Argument 2, imagine that (contrary to fact) both premises were true. IF that were the case, then it would have to be the case that the conclusion was true also. So, as you've noticed, a valid argument can have a false conclusion. That's not a shocking fact—it just follows from the way logicians define 'validity.'

Some of my students have been quite upset about the idea that arguments like argument 2 are valid. I apologize to them, but I try to reassure them that they're not being tricked into believing something absurd. That's just a consequence of the damned definition! What they're thinking of, I'd guess, is what logicians call a 'sound' argument—a valid argument *with all true premises*. Of course, this would also have a true conclusion.

Here, for the sake of completeness, is an invalid argument:

### **ARGUMENT 3**

P: The test is on Monday or Wednesday.

P: It's not on Thursday.

C: It's on Wednesday.

In this case, obviously, you can contemplate a possible state of affairs in which both premises are true, but the conclusion is false (because the test is on Monday).

A picturesque way of thinking about validity, as defined above, uses the notion of a *possible world*. This is merely a way things might be—that is, a logically consistent set of facts. Possibility, in the logicians' sense, is a broader notion than the ordinary one. A possible world can include flying pigs, a planet the size of earth made totally of chocolate fudge, perpetual motion machines, and so on. There are, obviously, a large number of possible worlds.

So we can define validity in terms of possible worlds: an argument is valid just in case there is no possible world in which the premises are all true, but the conclusion is false. (Have a look at Arguments 1 and 2, and convince yourself that there's no possible world like that, in either case.) But an argument is invalid otherwise—that is, if there is at least one possible world in which the premises are true and the conclusion is false. Examine Argument 3 with this in mind.

You might be wondering at this point why logicians define validity in this perverse way, which allows perfectly useless arguments like Argument 2 to be called valid. Isn't the point of

logic to distinguish arguments that get us at the truth from those that don't?

Well, yes, in a way. Logicians do aim at this. A variety of argument that's useful in this way is a sound argument—a valid one with true premises. The problem is that logicians are experts only on validity—they can't tell you any better than anyone else whether the premises are true or not. That's the job of some other science. Logic can tell you that Arguments 1 and 2 are valid, but to find out (if you didn't already know it) that the premises of 1 are true and the premises of 2 aren't, you have to go to an expert on the facts about birds, pigs, and snakes.

### *A Deeper Perversion of Logicians*

THE FACT THAT ARGUMENT 2 is counted as valid might be somewhat disturbing, but the following might seem more deeply perverse. Take a look at

#### **ARGUMENT 4**

P: All birds have wings.

P: Some birds don't have wings.

C: THEREFORE pigs are reptiles.

Is this argument valid? To find this out, remember, you're not supposed to worry about whether anything is actually true or false. You're supposed to ask yourself: is there a possible world in which both premises are true and the conclusion is false? If YES, then it's invalid. If NO, then it's valid. Think carefully about this for a while, and then look at the answer just below.

The answer is NO: there is no possible world in which the premises are both true, but the conclusion is false. Why not? Because those premises contradict each other: if one is true, the other has to be false. So there isn't any possible world in which they're both true. (So, of course, there isn't any possible world in which they're both true and the conclusion is false.) The answer is NO, so argument 4 is valid.

Students who got upset about the validity of Argument 2 got really upset about the validity of Argument 4. "But they always told us in high school," they object, "that a valid argument is one in which *the conclusion follows from the premises*. It clearly doesn't in Argument 4."

Right, it doesn't, but that's not the definition of validity logicians have been using for about a hundred years. Get with it, kids! The most recent logic your high-school logic teacher knew is a hundred years old! That old-fashioned definition is no good because there isn't any good way to determine what "follows from" premises and what doesn't.

Notice, before we leave this example, that this kind of argument—with contradictory premises, *can't be* sound, so it must be useless.

**TWO QUESTIONS TO THINK ABOUT:**

- a) Why can't arguments with contradictory premises be sound?
  
- b) Some statements are logically true—that is, they are true in all possible worlds. The statement 'All ducks are ducks' is one of them, and 'Every pig lives in Nebraska, or else at least some of them don't' is another. Another equally perverse consequence of this definition of validity is that every argument with a logically true premise is valid. Here's one:

**ARGUMENT 5**

P: Pigs can fly.

P: It snows in Vermont.

C: THEREFORE all ducks are ducks.

See if you can use a kind of reasoning similar to the one we used on Argument 4 to explain why this argument is valid.