3.4 You're Wrong (p. 83)

A Frequent Flier Bonus

Well, how *do* you figure out the odds of a crash among one million flights? Let's look first at the simpler jelly bean case, second selection procedure.

When previous picks are replaced, there are four different possibilities for the results of two picks:

PICK 1	PICK 2
R	R
R	G
G	R
G	G

Each of these possibilities is equally probable. Now, among these four ways, one has *no* red jelly beans picked (the last one on the list). That means that the chances of your picking no red jelly beans at all is ¼; and the chance of your picking the red jelly bean at least once is 1 minus ¼ = ¾.

How can we calculate these numbers in general? Note that the chance that any one pick does not get a red jelly bean is ½. The chance that the jelly bean picked on the first pick is non-red *and* the jelly bean picked on the second pick is non-red ½ X ½ = ¼. So the probability of getting red on at least one pick is 1 minus this: ¾.

Let's apply this to the airplane case. The chances of *no* crash on any one flight is 999,999/1,000,000. The chances of no crash during a million flights is this number times itself one million times, in other words, this number to the millionth power. In case you don't have the time to work out this arithmetic by hand, I have done it for you on my computer. The probability of no crash during a million flights is .363; so the probability that there will be at least one crash is .627—about two out of three. So it's more likely than not that at least one of these flights will end in a crash, but at least it's not certain. That's a small frequent flier bonus.

You needn't start worrying about the odds of being in a plane crash getting as high as two out of three, by the way. If you went on ten separate plane trips a day, every day of the year, it would take you 274 years to travel on a million flights.

A QUESTION TO THINK ABOUT: The chances that someone has a bomb in his luggage on any particular flight are small, but they're large enough to make some people worry. The chances that any particular flight is carrying *two* people who have bombs in their luggage are very much smaller. (If the probability that there's one person is $1/n$, then the probability that there are two people is $1/n^2$. Do you see why?)

The reasoning so far is correct, but consider the following. Smedley is quite worried that flights he's on will be destroyed by luggage-bombs. But the chances of there being *two* bombs are so small that he's not concerned about that event. What he does, then, is to carry a bomb in his luggage, designed not to go off, or course. He reasons that in order to be in danger of being killed by a luggage-bomb, someone else must have a bomb on the plane too, but the chances of two bombs on the same plane are so small that he doesn't have to worry. Smedley is making a mistake in his reasoning about probabilities. Can you explain exactly where his mistake is?[1]

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1 An old story told again by John Allen Paulos in *Innumeracy*, pp. 33–34.