Games – Exercises (All Answers)

Exercises for Chapter 7 of Steinhart, E. (2017) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 1)

1. Matched Pennies

Alice and Bob each have a big bag of pennies. They are standing on opposite sides of a table. They play a game. On each round of the game, each puts a penny down on the table at the same time. Neither player can see the other's penny in advance. Their strategies could be random or defined by some rule. After they put their pennies down, they look at the pennies. If both pennies show the same side (both heads, or both tails), then Alice gets both pennies. But if the pennies show different sides, then Bob gets both pennies. They can play as many rounds as they like. Draw the payoff matrix.

		Bob	
		Heads	Tails
Alice	Heads	-1 1	1 -1
	Tails	1 -1	-1 1

If they both put down Heads, then Alice keeps the penny she put down and she gains Bob's penny, for a net gain of 1; but Bob loses the penny he put down, for net loss of -1. Analogous reasoning yields the other payoffs.

2. Goalie & Kicker

A soccer goalie faces a kicker. A referree will blow a whistle, at which point the kicker kicks and the goalie jumps. They move at the same time. The kicker can kick the ball to his right or left; the goalie can jump to his right or left. Note that, since they face each other, the directions of their bodies are opposites. If the kicker kicks to his right and the goalie moves to his right, then they will have moved in opposite directions. Thus a goal will be scored. If a goal is scored, the kicker gets 1 and the goalie 0. But if the goalie blocks the kick, then the goalie gets 1 and the kicker gets 0. Draw the payoff matrix.

		Goalie	
		Left	Right
Kicker	Right	1 0	0 1
	Left	0 1	1 0

3. Battle of the Sexes

Ashley and Bailey are in love! They need to coordinate for their date tonight. Ashley would prefer to go to a movie, while Bailey would prefer to go to the opera. Tragically, they've lost their phones, and they have no way to communicate. All either one can do is show up at some place: each can go to either the movie theatre or to the opera house. If either gets their preference, that's a payoff of 2; if they don't get their preference, they get a payoff of only 1. But love is the most important thing! If they don't spend the evening together, their payoff is each 0. Draw the payoff matrix.

		Bailey	
		Movie	Opera
Ashley	Opera	0 0	2 1
	Movie	1 2	0 0

4. Chicken

Ali and Blue are racing towards each other in cars down a one-lane road. But up ahead, at the place where their paths converge, the road briefly widens out to two lanes. At that place, each must swerve either left or right. If they collide, the payoff is -10 for each. If they miss each other, it's 0 for each. Draw the payoff matrix.

		Blue	
		Left	Right
Ali	Right	-10 -10	0 0
	Left	0 0	-10 -10

Compare this with the kicker & goalie game (the soccer game). In the soccer game, coordinating means the goalie wins while not coordinating means the kicker wins. But in chicken, coordinating means both win while not coordinating means both lose.

5. Stag Hunt

Show that there is no strictly dominant strategy in the Stag Hunt. Since the payoff matrix is symmetrical, the calculation only needs to be done for player A. To show the failure of strict dominance, show both that $Stag_A$ *does not* strictly dominate $Hare_A$ and that $Hare_A$ *does not* strictly dominate $Stag_A$.

$Stag_A$ strictly dominates $Hare_A$
if and only if
$((U_A(Stag_A, Hare_B) > U_A(Hare_A, Hare_B))$ and
$(U_A(Stag_A, Stag_B) > U_A(Hare_A, Stag_B))$;
if and only if
$((0 > 3)$ and $(5 > 3)$;
if and only if
((false) and (true))
if and only if (false).

$Hare_A$ strictly dominates $Stag_A$
if and only if
$((U_A(Hare_A, Stag_B) > U_A(Stag_A, Stag_B))$ and
$(U_A(Hare_A, Hare_B) > U_A(Stag_A, Hare_B))$;
if and only if
$((3 > 5)$ and $(3 > 0))$
if and only if
((false) and (true))
if and only if (false).

$Stag_A$ *does not* strictly dominate $Hare_A$ and $Hare_A$ *does not* strictly dominates $Stag_A$. Therefore, there is no strictly dominant strategy for A in this game.

6. Farmers and their Crops

Two farmers control the local wheat market – they grow all the wheat in the area. After the harvest, each independently sets a price for their wheat. The price is either Low or High. where each can offer to sell High or Low. The payoff matrix is shown below.

Farmer Alice \ Farmer Bob	Low	High
Low	1 1	0 4
High	4 0	2 2

Look for a Nash equilibrium by filling in Table 1. For each blank cell in the table, fill in the change in payoff (going up or down or staying the same), and state whether person in the column will change their strategy.

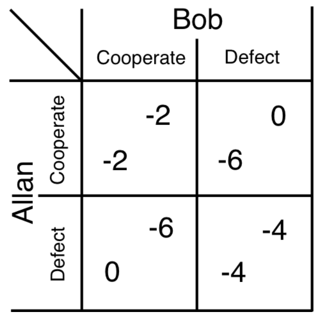
Choice of Alice	Choice of Bob	Ask Alice about changing	Ask Bob about changing
High	High	If Alice changes to Low, her payoff goes *up* from 2 to 4. Change to Low.	If Bob changes to Low, his payoff goes *up* from 2 to 4. Change to Low.
High	Low	If Alice changes to Low, then her payoff goes *up* from 0 to 1. Change to Low.	If Bob changes to High, then his payoff goes *down* from 4 to 2. Stay at Low.
Low	High	If Alice changes to High, then her payoff goes *down* from 4 to 2. Stay at Low.	If Bob changes to Low, then his payoff goes *up* from 0 to 1. Change to Low.
Low	Low	If Alice changes to High, then her payoff goes *down* from 1 to 0. No change.	If Bob changes to High, then his payoff goes *down* from 1 to 0. No change.

Table 1. Nash equilibrium in the Wheat Market?

The table shows that (Low_A, Low_B) is a Nash equilibrium: any strategy profile (x_A, x_B) changes to (Low_A, Low_B).

7. Prisonner’s Dilemma – Nash Equilibrium.

Below is the payoff matrix for the Prisonner’s Dilemma.



Look for a Nash equilibrium by filling in Table 2. For each blank cell in the table, fill in the change in payoff (going up or down or staying the same), and state whether person in the column will change their strategy.

Choice of Allan	Choice of Bob	Ask Allan about changing	Ask Bob about changing
Coop	Coop	If Allan changes to Defect, his payoff goes *up* from -2 to 0. Change to Defect.	If Bob changes to Defect, his payoff goes *up* from -2 to 0. Change to Defect.
Coop	Defect	If Allan changes to Defect, then his payoff goes *up* from -6 to -4. Change to Defect.	If Bob changes to Coop, then his payoff goes *down* from 0 to -2. Stay at Defect.
Defect	Coop	If Allan changes to Coop, then his payoff goes *down* from 0 to -2. Stay at Defect.	If Bob changes to Defect, then his payoff goes *up* from -6 to -4. Change to Defect.
Defect	Defect	If Allan changes to Coop, then his payoff goes *down* from -4 to -6. No change.	If Bob changes to Coop, then his payoff goes *down* from -4 to -6. No change.

Table 2. Nash equilibrium in the Prisonner’s Dilemma?

The table shows that (Defect_A, Defect_B) is a Nash equilibrium: any strategy profile (x_A, x_B) changes to (Defect_A, Defect_B).