

# Information Theory – Exercises (No Answers)

Exercises for Chapter 6 of Steinhart, E. (2017) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. Copyright (C) 2017 Eric Steinhart. Non-commercial educational use encouraged! All others uses prohibited. (Version 2)

6.1 Compute the following:

$$\log_3 9 \qquad \log_3 81 \qquad \log_{10} 100 \qquad \log_{10} 0.01$$

6.2A Construct the Huffman tree for these symbols and associated probabilities.

Command	Sit	Stand	Left	Right
Probability	7/12	3/12	1/12	1/12

6.2B Write down the binary codes for the symbols for the tree in 7.2A.

6.2C Calculate the average information of the code in 7.2B.

6.3A Construct a Huffman tree for these symbols and associated probabilities:

Command	Sit	Stand	Kneel	Left	Right	Up	Down
Probability	1/3	1/6	1/6	1/12	1/12	1/12	1/12

6.3B Write down the binary codes for the symbols for the tree in 7.3A.

6.3C Calculate the average information of the code in 7.3B.

6.4 The pie charts below illustrate the relative probabilities of the four values of a variable. That is, the area in each slice of pie represents the probability of the value. Which pie chart has the lowest entropy? Which has the highest entropy?

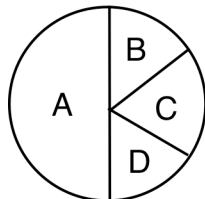


Chart 1

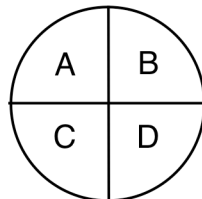


Chart 2

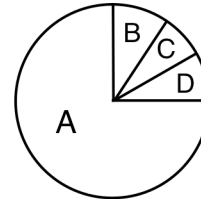


Chart 3

6.5 Calculate the Shannon entropy of a fair 8-sided die.

6.6 Calculate the Shannon entropy of the following alphabet:

a	e	i	u	p	t	k	w
1/4	1/8	1/8	1/8	1/8	1/8	1/16	1/16

6.7 Calculate the Shannon entropy of the following alphabet:

a	e	i	u	p	t	k	w
1/4	1/16	1/16	1/8	1/4	1/16	1/16	1/8

6.8 Calculate the entropy of the weather in Table 6.10. The calculation follows the same logic as the calculation of the entropy of the activity in Table 6.10. To calculate the entropy of the weather, it helps to use a scientific calculator or a spreadsheet like Excel.

6.9 Bob records his activities and the weather on the same days as Alice. Out of 256 days: 90 are sunny, 76 are cloudy, and the remaining 90 are rainy. On any day, Bob either jogs, does yoga, or just sits. He correlates his activities with the weather in Table 6X.1 (which resembles Alice's Table 6.10). The variable X ranges over his activities while the variable Y ranges over the weather. Note that Bob's activities are not correlated with the weather at all; they are random.

P(X,Y)		X			P <sub>M</sub> (Y)
		jogs	yoga	sits	
Y	sunny	$\frac{30}{256}$	$\frac{30}{256}$	$\frac{30}{256}$	$\frac{90}{256}$
	cloudy	$\frac{25}{256}$	$\frac{26}{256}$	$\frac{25}{256}$	$\frac{76}{256}$
	rainy	$\frac{30}{256}$	$\frac{30}{256}$	$\frac{30}{256}$	$\frac{90}{256}$
P <sub>M</sub> (X)		$\frac{85}{256}$	$\frac{86}{256}$	$\frac{85}{256}$	1

**Table 6X.1** A joint probability distribution P(activity, weather) for Bob.

6.9A Calculate the entropy of Bob's activity.

6.9B Calculate the entropy of the weather.

6.9C Calculate the conditional entropy H(X|Y).

6.9D Calculate the mutual information I(activity; weather) for Bob.

6.9E Carefully compare the case of Bob with the case of Alice.

6.10 Stacy records her activities and the weather on the same days as Alice. Out of 256 days: 90 are sunny, 76 are cloudy, and the remaining 90 are rainy. On any day, Stacy either jogs, does yoga, or just sits. She correlates her activities with the weather in Table 6X.2 (which resembles Alice's Table 6.10). The variable X ranges over her activities while the variable Y ranges over the weather. Note that Stacy's activities are perfectly correlated with the weather.

P(X,Y)		X			P <sub>M</sub> (Y)
		jogs	yoga	sits	
Y	sunny	$\frac{90}{256}$	$\frac{0}{256}$	$\frac{0}{256}$	$\frac{90}{256}$
	cloudy	$\frac{0}{256}$	$\frac{76}{256}$	$\frac{0}{256}$	$\frac{76}{256}$
	rainy	$\frac{0}{256}$	$\frac{0}{256}$	$\frac{90}{256}$	$\frac{90}{256}$
P <sub>M</sub> (X)		$\frac{90}{256}$	$\frac{76}{256}$	$\frac{90}{256}$	1

**Table 6X.2** A joint probability distribution P(activity, weather) for Stacy.

6.10A Calculate the entropy of the activity. The entropy of the activity is the same as in exercise 6.8.

6.10B Calculate the entropy of the weather. The entropy of the weather is the same as in exercise 6.8.

6.9C Calculate the conditional entropy H(X|Y). Here are the intermediate calculations:

6.9D Calculate the mutual information I(activity; weather) for Stacy.

6.9E Carefully compare the case of Bob with the case of Alice.

6.10 One of the striking features of mind-body dualism, theism, and telepathy is that advocates of those positions never talk about them using information theory. Why not?

6.11 It can be shown that  $H(X) - H(X|Y) = H(Y) - H(Y|X)$ . Use this and the Chain Rule to show that the definition of I(X;Y) in terms of joint entropy is equivalent to the definition of I(X;Y) in terms of conditional entropy.