Probability – Exercises (All Answers)

Exercises for Chapter 5 of Steinhart, E. (2017) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 2)

1. Simple Probabilities

A fair coin is tossed three times.

{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

What is the probability that exactly one tails appears? 3/8

What is the probability that the tosses produce exactly two heads? 3/8

Consider a standard deck of cards from which a card is chosen at random.

What is the probability it is a face card? 12/52 = 3/13.

What is the probability it is not a face card? 40/52 = 10/13.

An urn contains 15 white balls, 35 red balls, and 50 black balls.

What is the probability of drawing a red ball? 35/100.

What is the probability of drawing either a white or red ball? 50/100

A combination lock has three numerical dials, each of which shows a number from 0 through 9. Exactly one combination opens the lock.

What is the probability that the combination starts with 6? 1/100

What is the probability that the combination is less than 500? 1/2

Consider a standard tic-tac-toe board. X goes first and moves randomly.

What is the probability X marks the center box? 1/9

What is the probability X marks a corner? 4/9

2. Conditional Probabilities

Consider the following matrix of numbers of objects in a box.

	Red	**White**	**Black**	**Totals**
Cube	10	10	80	100
Sphere	10	20	70	100
Tetrahedron	10	30	60	100
Totals	30	60	210	300

You randomly select an item from the box.

What is the probability of selecting a black cube? 80/300 = 8/30 = 4/15

What is the probability of selecting a white sphere? 20/300 = 1/15

What is P(red or white)? = P(red) + P(white) = 90/300.

What is P(cube or sphere)? = P(cube) + P(sphere) = 200/300 = 2/3.

What is P(white | sphere)? = P(white & sphere) / P(sphere) = 20/100.

What is P(red | cube)? = P(red & cube) / P(cube) = 10/100.

What is P(cube | red) ? = P(red & cube) / P(red) = 10/30.

What is P(cube | black) ? = P(black & cube) / P(black) = 80/210.

3. Conditional Probabilities

Predator	0	0	1	0	0	1	0	0	1	1	0	1	0	0	1	1	0	0	1	0
Detector	0	1	1	1	0	1	0	0	1	1	1	1	0	0	1	0	1	0	0	0

P(there is a predator) = 8/20

P(there is no predator) = 12/20

P(the detector is on) = 10/20

P(the detector is off) = 10/20

P(there is a predator & the detector is on) = 6/20

P(there is no predator & the detector is on) = 4/20

P(there is a predator & the detector is off) = 2/20

P(there is no predator & the detector is off) = 8/20

P(there is a predator | the detector is on)

= P(there is a predator & the detector is on) / P(the detector is on)

= (6/20) / (10/20) = 6/10

P(there is no predator | the detector is on)

= P(there is no predator & the detector is on) / P(the detector is on)

= (4/20) / (10/20) = 4/10

P(there is a predator | detector is off) =

= P(there is a predator & detector is off) / P(the detector is off)

= (2/20) / (10/20) = 2/10

P(there is no a predator | the detector is off) =

= P(there is no predator & detector is off) / P(the detector is off)

= (8/20) / (10/20) = 8/10

4. Bayes Theorem

	In Matrix = M	In Reality = ~M	Total
Déjà vu = D	95	100	195
No déjà vu = ~D	5	99800	99805
Total	100	99900	100000

Given the table above, calculate the probability that you're in the matrix given that you have déjà vu.

P(M) = 100/100000

P(D & M) = 95/100000

P(D | M) = P(D & M) / P(M) = 95/100.

P(D) = 195/100000.

P(M | D) = (P(D | M) * P(M)) / P(D)

P(M | D) = ((95/100) * (100/100000)) / (195/100000) = 95/195.

5. Bayes Theorem

Bayes theorem can help you infer a cause from an effect, given that you know something about the effects produced by the possible causes. Consider a chess tournament in which the player is the cause and the first move is an effect.

You are at a tournament where you cannot see the person who is playing white. You know it could be Fischer, Spassky, or Tal. You know the probabilities that the first move is d4 or e4 given these three players. For instance, P(d4 | F) is the probability that the first move is d4 given Fischer is playing white. You know all of the following:

$$P(\,d4 \mid F) = \frac{5}{100}\;; \qquad P(\,d4 \mid S) = \frac{85}{100}\;; \qquad P(\,d4 \mid T) = \frac{70}{100}\,;$$

$$P(\,e4 \mid F) = \frac{95}{100}\;; \qquad P(\,e4 \mid S) = \frac{15}{100}\;; \qquad P(\,e4 \mid S) = \frac{30}{100}\,.$$

And you know the odds are equal that Fischer, Spassky, or Tal are playing white, which means you know that:

$$P(F) = P(S) = P(T) = \frac{1}{3}\,.$$

And you know that $P(d4) = \frac{160}{300}$ while $P(e4) = \frac{140}{300}$.

Suppose the first move played is e4. Use Bayes theorem to answer the following:

What is the probability that the player is Fischer?

$$P(F \mid e4) = \frac{P(e4 \mid F) * P(F)}{P(e4)} = \frac{95}{140}$$

What is the probability that the player is Spassky?

$$P(S \mid e4) = \frac{P(e4 \mid S) * P(S)}{P(e4)} = \frac{15}{140}$$

Suppose the first move played is d4. Use Bayes theorem to answer the following:

What is the probability that the player is Spassky?

$$P(S \mid d4) = \frac{P(d4 \mid S) * P(S)}{P(d4)} = \frac{85}{160}$$

What is the probability that the player is Tal?

$$P(T \mid d4) = \frac{P(d4 \mid T) * P(T)}{P(d4)} = \frac{70}{160}$$