Sets – Exercises (Selected Answers)

Exercises for Chapter 1 of Steinhart, E. (2017) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 2)

1. Collections

Write out the following:

The set of A:

The set of the set of A:

The set of A and B: **{A, B}**

The set of both A and the set of A: **{A, {A}}**

The set of A, B, and C:

If x is {A, B} and y is {C, D} then write out:

{x} = **{{A, B}}**

{x, y} =

{{x}} =

{{x}, y} = **{{{A,B}}, {C, D}}**

Answer the following (true or false):

1 = {1}? **False**

{1} = {{1}}?

{1, 1} = {1, {1}}?

{1, B, 2} = {2, 1, B}? **True**

{A, A} = {A}? **True**

{A, A} = {{A}}?

2. Membership

True or false:

Is A ∈ {A}?

Is {A} ∈ {{A}}? **True**

Is A ∈ {{A}}? **False**

Is {B} ∈ {{A}, {B}}?

Is {A, B} ∈ {A, B}?

Is {} ∈ {A}? **False**

3. Set Builders

Using the set Y = {1, A, 2, B, 3, C}, write out the following sets:

$\{ x \in Y \mid x \text{ is a letter}\}$ =

$\{ x \in Y \mid x \text{ is a number}\}$ = **{1, 2, 3}**

If X = {A, B, C, 1, 2, 3}, Y = {A, B, C}, and Z = {1, 2, 3}, then write:

$\{x \in X \mid x \text{ is also in } Y\}$ = **{A, B, C}**

$\{x \in X \mid x \in Z\}$ =

4. Unions

Write out:

$\{a, b\} \cup \{1\}$ = **{a, b, 1}**

$\{a, b\} \cup \{c, d\}$ =

$\{a\} \cup \{b\} \cup \{c\}$ =

$\{\{A\}\} \cup \{\{B\}\}$ =

$\{a\} \cup \{\{1\}\}$ = **{a, {1}}**

$\{A\} \cup \{\{A\}\}$ = **{A, {A}}**

5. Intersections

$\{a, b\} \cap \{a\}$ =

$\{a, b\} \cap \{\{a\}\}$ =

$\{1, 2, 3\} \cap \{a, b\}$ = **{}**

$\{\{\}, A\} \cap \{A, \{\}\}$ = **{{}, A}**

$\{\{\}, 2, 3\} \cap \{\{\}\}$ =

$\{a, b, c\} \cap \{b, c, d\}$ = **{b, c}**

6. Subsets

True or false.

{A, B} is a subset of {A, B, C}? **True**

{A} is a subset of {A, B}? **True**

A is a subset of {A}?

{A} is a subset of {{A}}? **False**

{A} is a subset of {A, {A}}?

{A, B} is a subset of {A, B}?

Write the subsets of {1, 2}:

7. Rank

Assume that A, B, and C are individuals on rank 0.

rank of {A} = **1**

rank of {A, {A}} = **2**

rank of {{{B}}, {C}} =

rank of {{A, B}, {C}} =

8. Power Sets

The power set of {1, A} is:

The power set of {Q} is: **{{}, {Q}}**

The power set of {{}} is:

Write the power set of the power set of {A}:

the power set of {A} =
the power set of {{}, {A}} =

9. Some Transformations of Sets

Suppose X = {A, B, C}.
Write the set that results from replacing each $x \in$ X with {x}:

{{A}, {B}, {C}}

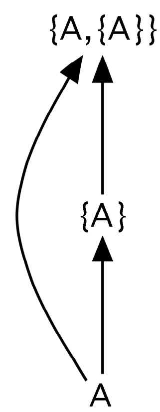
Suppose X = {{{A}, {B}}, {{C}}}.
Write the set that results from replacing each $x \in$ X with $\cup x$:

10. Diagramming Sets

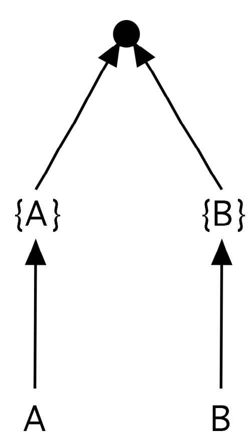
Use names or dots for sets and an arrow from *x* to *y* iff *x* is a member of *y*.

Draw the diagram for {A, B}.

Draw the diagram for {A, {A}}



Draw the diagram for {{A}, {B}}.



Draw the diagram for {{}, {{}}}.

11. Sets and Selections

Fill in the table with 0s and 1s to express all selections. Write the selected set in the rightmost cell of each row.

{}	A	{A}	
1	1	1	
1	1	0	{{}, A}
1	0	1	
1	0	0	{{}}
0	1	1	{A, {A}}
0	1	0	
0	0	1	
0	0	0	{}

Now write the set of all sets from the rightmost cell of each row:

12. Numbers as Sets

Using the idea that *n* is the set of all numbers less than *n*, write out:

0 = **{}**

1 =

2 =

3 = **{0, 1, 2}**

4 =

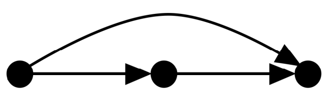
13. Diagramming Numbers as Sets

Draw a diagram for each of the numbers in exercise 12 above:

0 

1

2



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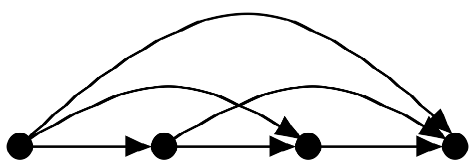

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14. Iteration versus Accumulation

An *iterative hierarchy* says that every next level is just the power set of the previous level. Consider this iterative hierarchy:

H(0) = {A}; H(n+1) = pow H(n).

Write out levels H(0), H(1), and H(2) of this iterative hierarchy:

H(0) =

H(1) = **{{}, {A}}**

H(2) =

A *cumulative hierarchy* says that every next level is the power set of the previous level unioned with the previous level. Consider this cumulative hierarchy:

K(0) = {A}; K(n+1) = pow K(n) $\cup$ K(n).

Write out levels K(0), K(1), and K(2) of this cumulative hierarchy:

K(0) = {A}

K(1) = {{}, A, {A}}

Write out pow K(1). (How can exercise 11 help you?)

K(2) =

Give an example of an object that appears on K(1) but not on H(1): **A**

List all objects that appear on K(2) but not on H(2):

A, {A}, {A, {A}}, {{}, A}, {{}, A, {A}}

Explain why K(n+1) is richer than H(n+1) for $n > 0$:

Because both members and subsets of K(n) appear on K(n+1), while only subsets of H(n) appear on H(n+1).

15. Ordered Pairs

Diagram (Sue, Bob)

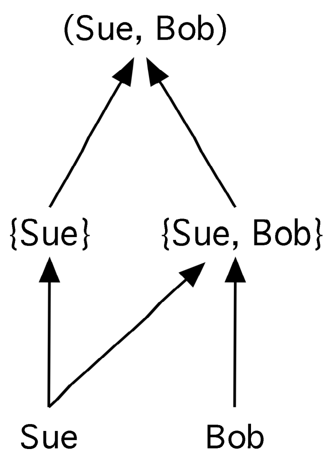

Diagram (Bob, Bob)

Diagram (Sue, {Sue})

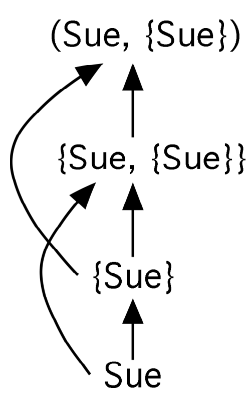

Diagram ({}, {{}})

16. Cartesian Products

Write the Cartesian Product $\{A, B\} \times \{1, 2\}$.

Write the Cartesian Product $\{\text{Abe}, \text{Bob}, \text{Sue}\} \times \{\text{Happy}, \text{Sad}\}$.