Relations – Exercises

Exercises for Chapter 2 of Steinhart, E. (2009) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 1)

1. Relations

Table 1 shows part of the is-a-parent-of relation for an imaginary family:

Parent	*Children*
Sally	Abe, Bob, Mary
Abe	Sue, Jill
Mary	Diane, Kathy, Rachel

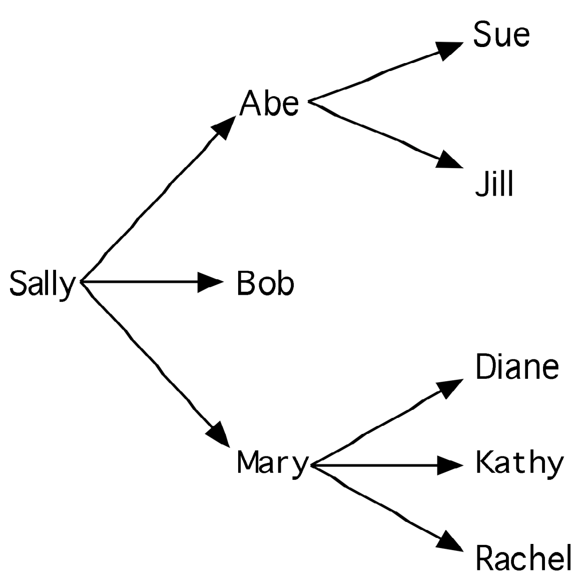
Table 1. Part of the is-a-parent-of relation.

Write the relation in the table as a set of ordered pairs. The set includes (x, y) iff x is a parent of y. Thus:

{ **(Sally, Abe), (Sally, Bob), (Sally, Mary),**
(Abe, Sue), (Abe, Jill),
(Mary, Diane), (Mary, Kathy), (Mary, Rachel) }

2. Graphs of Relations

For any (x, y) in a relation R, the *graph* of R has an arrow from x to y. Draw the graph of the is-a-parent-of relation from Table 1 above:



3. Recursion and Generations

An *original ancestor* is someone in Table 1 who does not have a parent in Table 1. G(0) is the set of original ancestors from Table 1. Write G(0) as a set:

G(0) = **{Sally}**

G(1) includes every child of any person in G(0). Write G(1) as a set:

G(1) = **{Abe, Bob, Mary}**

G(2) includes every child of any person in G(1). Write G(2) as a set:

G(2) = **{Sue, Jill, Diane, Kathy, Rachel}**

4. Recursion and Generations

H(0) includes the original ancestors from Table 1. Write H(0) as a set:

H(0) = **{Sally}**

Does H(0) = G(0)? **Yes**

Now let $C(x) = \{ y \mid y \text{ is a child of } x \}$.

Let $H(1) = \{ C(x) \mid x \in G(0)\}$. Write H(1) as a set:

H(1) = **{{Abe, Bob, Mary}}**

Does H(1) = G(1)? **No**

Let $H(2) = \{ C(x) \mid x \in G(1)\}$. Write H(1) as a set:

H(2) = **{{Sue, Jill}, {}, {Diane, Kathy, Rachel}}**

Does H(2) = G(2)? **No**

5. Ancestrals

Consider the relation *x is an improvement of y*. The ancestral of this relation is *x is better than y*. For example, if Gamma is an improvement of Beta, and Beta is an improvement of Alpha, then Gamma is better than Alpha. Use the formula for defining ancestrals to write the definition of x is better than y:

x is better than y iff
either x is an improvement of y
or there is some z such that x is an improvement of z and z is better than y

Consider the relation *x is linked to y*. The ancestral of this relation is *x is chained to y*. For example, if Gamma is linked to Beta, and Beta is linked to Alpha, then Gamma is chained to Alpha. Use the formula for ancestrals to write the definition of x is chained to y:

x is chained to y iff
either x is linked to y
or there is some z such that x is linked to z and z is chained to y

6. Functional Notation

Person	*Weight*
Socrates	150
Plato	180
Aristotle	200

Table 2. Weights of persons.

The function in this table maps persons onto their weights. It is W. Write:

W(Socrates) = **150**

W(Plato) = **180**

W(Aristotle) =**200**

Person	*Height*
Socrates	64
Plato	72
Aristotle	67

Table 3. Heights of persons.

The function in this table maps persons onto their heights. It is H. Write:

H(**Plato**) = 72

H(**Socrates**) = 64

H(**Aristotle**) = 67

7. Functions (There are many ways to answer this.)

Give a function (use arrows) from people to emotions.

Abe	Happy
Bob	Sad
Sue	

Write this function as a set of ordered pairs.

Write a many-1 function from students to grades.

Becky	A
Carl	B
Mike	C
Sue	D
Tim	F

Write a 1-1 function from students to grades.
Don't just draw arrows straight across.

Becky	A
Carl	B
Mike	C
Sue	D
Tim	F

8. Isomorphism

Consider the two situations shown as Situation 1 and Situation 2.

Situation 1.	Situation 2.
Socrates helps Theaetetus. Theaetetus produces an idea. Socrates examines the idea.	Sally helps Susan. Susan produces a baby. Sally examines the baby.

These two situations are isomorphic. Let f be a function from Situation 1 to Situation 2 that preserves the relational structure of the situations. Complete these:

f(**Socrates**) = Sally;

f(Theaetetus) = **Susan** ;

f(**idea**) = baby.

9. The Eternal Return of the Same

Suppose some universe is eternally recurrent both into the past and into the future. This universe divides into cosmic epochs. These epochs are isomorphic – each epoch is an exact qualitative duplicate of every other epoch. The epochs are numbered. The function C associates the integer n with the n-th epoch.

Any epoch contains some individuals. An individuating function for the individual X associates each integer n with the instance of X in the n-th epoch. Nietzsche uses his character Zarathustra to talk about eternal recurrence. Zarathustra has two animals, an eagle and a snake. They tell him that they understand his theory of eternal recurrence:

> Behold, we know what you teach: that all things recur eternally and we ourselves with them, and that we have already existed an infinite number of times before and all things with us. You teach that there is a great year of becoming, a colossus of a year: this year must, like an hour-glass, turn itself over again and again, so that it may run down and run out anew. So that all these years resemble one another, in the greatest things and in the smallest, so that we ourselves resemble ourselves in each great year, in the greatest things and in the smallest. And if you should die now, O Zarathustra: behold, we know too what you would then say to yourself . . . "Now I die and decay" you would say, "and in an instant I shall be nothingness. Souls are as mortal as bodies. But the complex of causes in which I am entangled will recur -- it will create me again! I myself am part of these causes of the eternal recurrence. I shall return, with this sun, with this earth, with this eagle, with this serpent -- not to a new life or a better life or a similar life: I shall return eternally to this identical and self-same life, in the greatest things and in the smallest, to teach once more the eternal recurrence of all things." (Nietzsche, 1978: III: 13/2)

Let's focus on three individuals in each epoch: Zarathustra, his eagle, and his serpent. The individuating function Z maps each integer n onto the instance of Zarathustra in epoch n. The individuating function for the eagle is E and the individuating function for the serpent is S. We thus have three individuating functions: Z, E, S. Now say

x *is a counterpart of* y iff
  there is some individuating function f such that
    for some integer n, and
      for some integer m,
        f(n) = x and f(m) = y.

Answer the following:

(**True**) (F) The counterpart relation is reflexive.

(**True**) (F) The counterpart relation is symmetric.

(**True**) (F) The counterpart relation is transitive.

(**True**) (F) The counterpart relation is an equivalence relation.

10 Bolzano's Degrees of Being

Bernard Bolzano was a 19th century German mathematician and philosopher. His book, *Paradoxes of the Infinite,* is the start of modern set theory and the modern theory of infinity. *Paradoxs of the Infinite* is filled with many examples of infinite structures. Bolzano says that infinity is illustrated by

> the fact that in all God's creation no degree of being is the highest and none the lowest; the further fact that in every degree however high, and at every instant however early, creatures have existed which have risen to that degree by their rapid progress; and the converse fact that in every degree however low, and at every instant however late, creatures will exist which despite continual progress have only then attained that degree. (Bolzano, 1950, sec. 58)

The integers are the set Z. That is, Z = {. . . –3, -2, -1, 0, 1, 2, 3, . . . }.

The set of creatures (at all places and times) is C.

Assume that time and the degrees of being both have the structure of the integers.

Write a function that expresses Bolzano's description in sec. 58.

The function associates every (time, degree) pair with some set of creatures. So the domain of the function is Z × Z. The codomain is the power set of C. Thus

f: Z × Z → pow C.

References

Bolzano, B. (1950) *Paradoxes of the Infinite*. Trans. F. Prihonsky. London: Routledge Kegan Paul.

Nietzsche, F. (1978) *Thus Spake Zarathustra*. Trans. R. J. Hollingdale. New York: Penguin Books.