

CHAPTER 9

Solutions to the Even-Numbered Questions in the Text

9.1

2. Logically consistent.
4. Logically consistent.
6. Logically inconsistent.

9.4

2. $C \leftrightarrow X$
T F F
4. $\neg(X \rightarrow \neg A)$
F F T FT
6. $\neg[(A \rightarrow X) \leftrightarrow X]$
F T F F T F
8. $A \rightarrow (\neg B \vee C)$
T T FT T T
10. $\neg A \leftrightarrow (A \leftrightarrow C)$
FT F T T T
12. $\neg[\neg A \& \neg(B \leftrightarrow \neg X)]$
T FT F F T T FT
14. $\neg(A \leftrightarrow B) \leftrightarrow \neg(X \leftrightarrow Z)$
F T T T T F F T F
16. $[\neg A \rightarrow (Z \leftrightarrow X)] \leftrightarrow [\neg X \rightarrow (A \& B)]$
FTT F T F T T T T T T T
18. $\neg[\neg A \leftrightarrow (\neg B \leftrightarrow \neg X)] \vee Z$
F FT T FT F T F F F
20. $\neg\{\neg[A \leftrightarrow (Z \leftrightarrow B)] \& \neg[Z \leftrightarrow (B \vee X)]\}$
T F T T F F T F T F F T T F

9.5

2. $D \& B$
4. $B \rightarrow D$
6. $\neg A \& (A \rightarrow D)$
8. $\neg C \rightarrow (D \& A)$
10. $(\neg A \vee \neg C) \rightarrow (\neg D \& \neg B)$
12. $B \rightarrow (\neg C \& \neg D)$
14. $A \leftrightarrow (B \& \neg D)$

9.6A

2.

A	B	$A \leftrightarrow B$	$\neg B$	$\neg A$	Path
T	T	T T T	F F	F T	M
T	F	T F F	T F	F T	M
F	T	F F T	F T	T F	M
F	F	F T F	T F	T F	T

There is a T path, so the set is consistent.

4.

H	Z	$H \& Z$	$Z \leftrightarrow H$	$\neg H$	Path
T	T	T T T	T T T	F T	M
T	F	T F F	F F T	F T	F
F	T	F F T	T F F	T F	M
F	F	F F F	F T F	T F	M

There are no T paths, so the set is inconsistent.

6.

A	B	C	$A \& B$	$B \leftrightarrow C$	$\neg A$	C	Path
T	T	T	T T T	T T T	F T	T	M
T	T	F	T T T	T F F	F T	F	M
T	F	T	T F F	F F T	F T	T	M
T	F	F	T F F	F T F	F T	F	M
F	T	T	F F T	T T T	T F	T	M
F	T	F	F F T	T F F	T F	F	M
F	F	T	F F F	F F T	T F	T	M
F	F	F	F F F	F T F	T F	F	M

There are no T paths, so the set is inconsistent.

8.

<i>M</i>	<i>N</i>	<i>R</i>	<i>M & N</i>	<i>N → R</i>	<i>R ↔ M</i>	<i>Path</i>
T	T	T	⊗ T ⊗	⊗ T ⊗	⊗ T ⊗	T
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

There is at least one T path, so the set is consistent.

10.

<i>H</i>	<i>B</i>	<i>R</i>	<i>H → B</i>	<i>¬B</i>	<i>¬B → R</i>	<i>¬R</i>	<i>Path</i>
T	T	T	⊗ T ⊗	⊗⊗	⊗⊗ T ⊗	⊗⊗	M
T	T	F	⊗ T ⊗	⊗⊗	⊗⊗ T ⊗	T⊗	M
T	F	T	⊗ F ⊗	T⊗	⊗⊗ T ⊗	⊗⊗	M
T	F	F	⊗ F ⊗	T⊗	⊗⊗ F ⊗	T⊗	M
F	T	T	F T ⊗	⊗⊗	⊗⊗ T ⊗	⊗⊗	M
F	T	F	F T ⊗	⊗⊗	⊗⊗ T ⊗	T⊗	M
F	F	T	F T ⊗	T⊗	⊗⊗ T ⊗	⊗⊗	M
F	F	F	F T ⊗	T⊗	⊗⊗ F ⊗	T⊗	M

There are no T paths, so the set is inconsistent.

9.6B

2. a. *M* b. *M ∨ A* c. *¬A*

<i>M</i>	<i>A</i>	<i>M</i>	<i>M ∨ A</i>	<i>¬A</i>	<i>Path</i>
T	T	T	⊗ T ⊗	⊗⊗	M
T	F	T	⊗ T ⊗	T⊗	T
F	T				
F	F				

There is at least one T path, so the set is consistent.

4. a. $H \leftrightarrow R$ b. $\neg H \& R$

H	R	$H \leftrightarrow R$	$\neg H \& R$	<i>Path</i>
T	T	F T F	FF F F	M
T	F	F F F	FF F F	F
F	T	F F F	FF T F	M
F	F	F T F	FF F F	M

There are no T paths, so the set is inconsistent.

6. a. $F \& (S \leftrightarrow T)$ b. $T \vee (\neg F \& \neg S)$

F	S	T	$F \& (S \leftrightarrow T)$	$T \vee (\neg F \& \neg S)$	<i>Path</i>
T	T	T	F T F F F	F T F F F F F	T
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

There is at least one T path, so the set is consistent.

8. a. $B \& \neg M$ b. $B \leftrightarrow R$ c. $R \leftrightarrow M$

B	M	R	$B \& \neg M$	$B \leftrightarrow R$	$R \leftrightarrow M$	<i>Path</i>
T	T	T	F F F F	F T F	F T F	M
T	T	F	F F F F	F F F	F F F	F
T	F	T	F T F F	F T F	F F F	M
T	F	F	F T F F	F F F	F T F	M
F	T	T	F F F F	F F F	F T F	M
F	T	F	F F F F	F T F	F F F	M
F	F	T	F F F F	F F F	F F F	F
F	F	F	F F F F	F T F	F T F	M

There is no T path, so the set is inconsistent.

10. a. $C \rightarrow K$ b. $\neg K$ c. $C \rightarrow W$ d. $\neg C \rightarrow W$

C	K	W	$C \rightarrow K$	$\neg K$	$C \rightarrow W$	$\neg C \rightarrow W$	Path
T	T	T	T T T	F F	T T T	F F T T	M
T	T	F	T T F	F F	T F F	F F T F	M
T	F	T	T F F	F F	T T T	F F T T	M
T	F	F	T F F	F F	T F F	F F T F	M
F	T	T	F T T	F F	F T T	F F T T	M
F	T	F	F T F	F F	F T F	F F F F	M
F	F	T	F T F	F F	F T T	F F T T	T
F	F	F	F T F	F F	F T F	F F F F	M

There is at least one T path, so the set is consistent.

9.7A

2.

A	B	A	$A \rightarrow B$	$\neg\neg A$	Path
T	T	T	T T T	T F F	T
T	F				
F	T				
F	F				

Since there is at least one T path, the set is consistent so $\{A, A \rightarrow B\}$ does not entail $\neg A$.

4.

A	B	Z	$A \& \neg B$	A	$\neg Z$	Path
T	T	T	T F F F	T	F F	M
T	T	F	T F F F	T	T F	M
T	F	T	T T F F	T	F F	M
T	F	F	T T F F	T	T F	T
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Since there is at least one T path, the set is consistent, so $\{A \& \neg B; A\}$ does not entail Z.

6.

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$\neg(A \rightarrow C)$	Path
T	T	T	T T T	T T T	F T T T	M
T	T	F	T T T	T F F	T T F F	M
T	F	T	T F F	F T T	F T T T	M
T	F	F	T F F	F T F	T T F F	M
F	T	T	F T T	T T T	F F T T	M
F	T	F	F T T	T F F	F F T F	M
F	F	T	F T F	F T T	F F T T	M
F	F	F	F T F	F T F	F F T F	M

Since there is no T path, the set $\{A \rightarrow B; B \rightarrow C; \neg(A \rightarrow C)\}$ is inconsistent. So $\{A \rightarrow B; B \rightarrow C\}$ entails $A \rightarrow C$.

9.7B

2.

A	B	$A \vee B$	$\neg B$	$\neg\neg A$	Path
T	T	T T T	F T	T F T	M
T	F	T T F	T F	T F T	T
F	T	F T T	F T	F T T	M
F	F	F F F	T F	F T T	M

There is one T paths, so the set is consistent, so the argument is invalid.

4.

A	B	C	$A \rightarrow (B \& C)$	$\neg(A \rightarrow C)$	Path
T	T	T	T T T T T	F T T T	M
T	T	F	T F T F F	T T F F	M
T	F	T	T F F F T	F T T T	F
T	F	F	T F F F F	T T F F	M
F	T	T	F T T T T	F F T T	M
F	T	F	F T T F F	F F T F	M
F	F	T	F T F F T	F F T T	M
F	F	F	F T F F F	F F T F	M

There are no T paths, so the set is inconsistent, so the argument is valid.

6.

M	T	B	$\neg M \vee (T \& B)$	$\neg M \leftrightarrow T$	$\neg T$	<i>Path</i>
T	T	T	FF T FFF	FF F F	FF	M
T	T	F	FF F FFF	FF F F	FF	F
T	F	T	FF F FFF	FF T F	TF	M
T	F	F	FF F FFF	FF T F	TF	M
F	T	T	FF T FFF	FF T F	FF	M
F	T	F	FF T FFF	FF T F	FF	M
F	F	T	FF T FFF	FF F F	TF	M
F	F	F	FF T FFF	FF F F	TF	M

There are no T paths, so the set is inconsistent, so the argument is valid.

8.

A	B	C	$\neg A \leftrightarrow (B \rightarrow C)$	$\neg A \& \neg C$	$\neg \neg B$	<i>Path</i>
T	T	T	FF F FFF	FF F FF	T FF	M
T	T	F	FF T FFF	FF F FF	T FF	M
T	F	T	FF F FFF	FF F FF	F FF	M
T	F	F	FF F FFF	FF F FF	F FF	M
F	T	T	FF T FFF	FF F FF	T FF	M
F	T	F	FF F FFF	FF T FF	T FF	M
F	F	T	FF T FFF	FF F FF	F FF	M
F	F	F	FF T FFF	FF T FF	F FF	M

There are no T paths, so the set is inconsistent, so the argument is valid.

10.

A	B	D	$\neg [A \vee (B \vee D)]$	$\neg B \rightarrow \neg A$	$\neg B$	<i>Path</i>
T	T	T	F F F FFF	FF T FF	FF	M
T	T	F	F F F FFF	FF T FF	FF	M
T	F	T	F F F FFF	FF F FF	TF	M
T	F	F	F F F FFF	FF F FF	TF	M
F	T	T	F F F FFF	FF T FF	FF	M
F	T	F	F F F FFF	FF T FF	FF	M
F	F	T	F F F FFF	FF T FF	TF	M
F	F	F	T F F FFF	FF T FF	TF	M

There are no T paths, so the set is inconsistent, so the argument is valid.

9.7C

2.

$$1. C \rightarrow R$$

$$2. \neg C$$

$$\therefore \neg R$$

C	R	$C \rightarrow R$	$\neg C$	$\neg\neg R$	Path
T	T	TTT	FT	TTT	M
T	F	TF	FT	FTT	F
F	T	FT	TF	TTT	T
F	F	FT	TF	FTT	M

There is a T path, so the set is consistent, so the argument is invalid.

4.

$$H \leftrightarrow C$$

$$C$$

$$\therefore H$$

H	C	$H \leftrightarrow C$	C	$\neg H$	Path
T	T	TTT	T	FT	M
T	F	TF	F	FT	F
F	T	FT	T	TF	M
F	F	FT	F	TF	M

There are no T paths, so the argument is valid.

6.

$$1. S \vee J$$

$$2. J \rightarrow W$$

$$\therefore S \vee W$$

S	J	W	$S \vee J$	$J \rightarrow W$	$\neg(S \vee W)$	Path
T	T	T	TTT	TTT	FTTT	M
T	T	F	TTT	TF	FTTT	M
T	F	T	TT	FT	FTTT	M
T	F	F	TT	FT	FTTT	M
F	T	T	FT	TT	FTTT	M
F	T	F	FT	TF	TFFF	M
F	F	T	FF	FT	FTTT	M
F	F	F	FF	FT	TFFF	M

There are no T paths, so the set is inconsistent, so the argument is valid.

8.

1. $S \rightarrow A$
2. $A \rightarrow W$

$\therefore S \rightarrow W$

S	A	W	$S \rightarrow A$	$A \rightarrow W$	$\neg(S \rightarrow W)$	Path
T	T	T	F T F	F T F	F F F F	M
T	T	F	F T F	F F F	T F F F	M
T	F	T	F F F	F T F	F F F F	M
T	F	F	F F F	F T F	T F F F	M
F	T	T	F T F	F T F	F F F F	M
F	T	F	F T F	F F F	F F F F	M
F	F	T	F T F	F T F	F F F F	M
F	F	F	F T F	F T F	F F F F	M

There are no T paths, so the argument is valid.

10.

1. $S \rightarrow A$
2. $A \rightarrow W$

$\therefore S \vee W$

S	A	W	$S \rightarrow A$	$A \rightarrow W$	$\neg(S \vee W)$	Path
T	T	T	F T F	F T F	F F F F	M
T	T	F	F T F	F F F	F F F F	M
T	F	T	F F F	F T F	F F F F	M
T	F	F	F F F	F T F	F F F F	M
F	T	T	F T F	F T F	F F F F	M
F	T	F	F T F	F F F	T F F F	M
F	F	T	F T F	F T F	F F F F	M
F	F	F	F T F	F T F	T F F F	T

There is a T path, so the argument is invalid.

9.7D

2.

A	B	$A \vee B$	$\neg A$	$\neg(?B)$	Path
T	T	F T F	F F		M
T	F	F T F	F F		M
F	T	F T F	T F		
F	F	F F F	T F		M

We want no T paths. Rows 1, 2, and 4 are not T paths immediately. So we need to make $\neg(?B)$ false when B is T to block that row. That means ?B is to be T when B is T. "B" will do the job.

4.

A	B	C	$(A \& C) \rightarrow B$	$C \& A$	$\neg(\neg B)$	Path
T	T	T	TTTTF	TTT		
T	T	F	TTFTT	FTT		M
T	F	T	TFFTT	FTT		M
T	F	F	TFFTF	FFT		M
F	T	T	FTTTF	FTT		M
F	T	F	FTFTT	FTT		M
F	F	T	FFTTF	FTT		M
F	F	F	FFTFF	FTT		M

We need to choose $\neg(\neg B)$ to block all T paths. But the two premises as they stand rule out all rows but row 1. So $\neg(\neg B)$ must be F in row 1, which means $\neg B$ must be T in row 1, when $\neg B$ is T. So “B” will work.

9.8A

2.

B	Q	$B \& Q$	$Q \leftrightarrow B$	Path
T	T	TTT	TTT	T
T	F	TF F	FTT	F
F	T	FTT	FTT	F
F	F	FFF	FTT	M

Answer: none

4.

Q	R	$Q \leftrightarrow R$	$\neg Q \leftrightarrow \neg R$	Path
T	T	TTT	TTT	T
T	F	TF F	FTT	F
F	T	FTT	FTT	F
F	F	FTT	FTT	T

Answer: equivalent

6.

A	B	C	$(A \& B) \& C$	$A \& (B \& C)$	Path
T	T	T	TTTTF	TTTTF	T
T	T	F	TTTF F	TF TFF	F
T	F	T	TF FT T	TF FT T	F
T	F	F	TF FF F	TF FF F	F
F	T	T	FT TT T	FT TT T	F
F	T	F	FT TF F	FT TF F	F
F	F	T	FT FT T	FT FT T	F
F	F	F	FT FF F	FT FF F	F

Answer: equivalent

8.

A	B	C	$\neg A \vee C$	$\neg B \& A$	Path
T	T	T	TTT	FTT	M
T	T	F	FTFF	FTT	F
T	F	T	FTT	FTT	T
T	F	F	FTFF	FTT	M
F	T	T	FTT	FTFF	M
F	T	F	FTFF	FTFF	F
F	F	T	FTT	FTFF	M
F	F	F	FTFF	FTFF	F

Answer: none

10.

A	B	C	$\neg A \leftrightarrow (B \vee C)$	$A \leftrightarrow (B \vee \neg C)$	Path
T	T	T	FT F TTT	T T TTT	M
T	T	F	FT F TTF	T T TTF	M
T	F	T	FT F FTT	T F FFT	F
T	F	F	FT T FFF	T T FFT	T
F	T	T	FT T TTF	F F TTF	M
F	T	F	FT T TTF	F F TTF	M
F	F	T	FT T FTT	F T FFT	T
F	F	F	FT F FFF	F F FFT	F

Answer: none

9.8B

2. a. $B \& J$
b. $B \& \neg J$

B	J	$B \& J$	$B \& \neg J$	Path
T	T	TT	TF	M
T	F	TF	TF	M
F	T	FT	FT	F
F	F	FF	FF	F

So these are contraries.

4. a. $H \& C$
 b. $C \rightarrow H$

H	C	$H \& C$	$C \rightarrow H$	<i>Path</i>
T	T	T T T	T T T	T
T	F	T F F	F T T	M
F	T	F F T	T F F	F
F	F	F F F	F T F	M

So none of these relations holds.

6. a. $A \& \neg S$
 b. $\neg A \vee (S \& F)$

A	S	F	$A \& \neg S$	$\neg A \vee (S \& F)$	<i>Path</i>
T	T	T	T F T	F T T T	M
T	T	F	T F T	F T F F	F
T	F	T	T T T	F F F T	M
T	F	F	T T T	F F F F	M
F	T	T	F F T	T T T T	M
F	T	F	F F T	T T F F	M
F	F	T	F F T	T T F T	M
F	F	F	F F T	T T F F	M

So these are contraries.

8. a. $\neg A \& \neg F$
 b. $(\neg A \vee \neg S) \vee \neg F$

A	F	S	$\neg A \& \neg F$	$(\neg A \vee \neg S) \vee \neg F$	<i>Path</i>
T	T	T	F F T	F F F F F	F
T	T	F	F F T	F T T T T	M
T	F	T	F F T	F F F T T	M
T	F	F	F F T	F T T T T	M
F	T	T	T F T	T T F T T	M
F	T	F	T F T	T T F T T	M
F	F	T	T T T	T T F T T	T
F	F	F	T T T	T T F T T	T

So none of these relations holds.

10. a. $A \& (S \& M)$
 b. $(\neg A \& \neg S) \vee S$

A	S	M	$A \& (S \& M)$	$(\neg A \& \neg S) \vee S$	Path
T	T	T	T T T	F F F T F	T
T	T	F	T F T F F	F F F T F	M
T	F	T	T F F F T	F F T F F	F
T	F	F	T F F F F	F F T F F	F
F	T	T			
F	T	F			
F	F	T			
F	F	F			

So none of these relations holds.

9.8C

2.

A	$A \rightarrow \neg A$	Path
T	T F F	F
F	F T F	T

Contingent

4.

A	B	$(A \& B) \rightarrow \neg A$	Path
T	T	T T T F F	F
T	F	T F F T F	T
F	T	F F T T F	T
F	F	F F F T F	T

Contingent

6.

A	B	$(A \& B) \rightarrow (B \& A)$	Path
T	T	T T T T T T T	T
T	F	T F F T F F T	T
F	T	F F T T T F F	T
F	F	F F F T F F F	T

Necessarily true

8.

A	B	$(A \rightarrow B) \rightarrow [(B \& A) \vee \neg(B \& A)]$	Path
T	T	T T T T T T T F T T T	T
T	F	T F F F T F F T T F F F T	T
F	T	F T T T T T F F T T F F F	T
F	F	F T T F T F F F F T T F F F	T

Necessarily true

10.

A	B	C	$A \rightarrow \{A \rightarrow [(A \vee B) \vee C]\}$	Path
T	T	T	T T T T T T T T T T	T
T	T	F	T T T T T T T T T F	T
T	F	T	T T T T T T T T T T	T
T	F	F	T T T T T T T T T F	T
F	T	T	F T F T F T T T T T	T
F	T	F	F T F T F T T T T F	T
F	F	T	F T F T F T F F F T T	T
F	F	F	F T F T F F F F F F	T

Necessarily true

9.8D

2. $F \& \neg F$

F	$F \& \neg F$	Path
T	T F F T	F
F	F F T F	F

Necessarily false

4. $R \rightarrow R$

R	$R \rightarrow R$	Path
T	T T T	T
F	F T F	T

Necessarily true

6. $(S \vee A) \vee \neg F$

<i>S</i>	<i>A</i>	<i>F</i>	$(S \vee A) \vee \neg F$	<i>Path</i>
T	T	T	T T T T F F	T
T	T	F	F T T T T F F	T
T	F	T	F T F T T F F	T
T	F	F	F T F T T F F	T
F	T	T	F T T T T F F	T
F	T	F	F T T T T F F	T
F	F	T	F F F F F F F	F
F	F	F	F F F F T T F	T

Contingent

8. $(F \leftrightarrow \neg\neg F) \& (\neg A \& \neg L)$

<i>F</i>	<i>A</i>	<i>L</i>	$(F \leftrightarrow \neg\neg F) \& (\neg A \& \neg L)$	<i>Path</i>
T	T	T	F T F F F F F F F F	F
T	T	F	F T F F F F F F F F	F
T	F	T	F T F F F F F F F F	F
T	F	F	F T F F F T T F F F	T
F	T	T	F T F F F F F F F F	F
F	T	F	F T F F F F F F F F	F
F	F	T	F T F F F F F F F F	F
F	F	F	F T F F F T T F F F	T

Contingent

10. $(H \& R) \leftrightarrow \neg(\neg H \& \neg R)$

<i>H</i>	<i>R</i>	$(H \& R) \leftrightarrow \neg(\neg H \& \neg R)$	<i>Path</i>
T	T	F T T T T F F F	T
T	F	F F F F T F F F F F	F
F	T	F F F F F T F F F F	F
F	F	F F F T F F F F F F	T

Contingent