Temporal Counterpart Theory

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1. Universe

A structure is an ordered *n*-tuple. A *universe* is a structure $(T, <, I, \delta, P_1, \ldots P_n, C)$. The items in this structure are:

- The item T is a set of times (instants). These are just numbers (we sometimes indicate that these numbers are times by associating them with the letter t, as in t_1).

- The item < is the usual order relation on T by magnitude. Thus earlier times are smaller numbers and later times are bigger numbers.

- The item I is the set of individuals. Each individual is temporally unextended and exists at exactly one time.

- The item δ associates each time in T with some individuals. These are the individuals that exist at time t.

- Each item P_i associates each time in T with the extension of the property P_i at that time. Each P_i is a synchronic property; it takes only individuals in the same time.

- The item C is the counterpart relation on the individuals in I.

At any time t, there is a *snapshot* of the universe. The snapshot is the structure of the universe at a single moment. The snapshot at t is $(\delta(t), P_1(t), \ldots P_n(t))$.

2. The Reference Function

For any NAME, the reference function f maps every (NAME, time) pair onto an individual. It should be clear that we can refer to individuals at times when they do not exist. For example, Plato no longer exists; but we can still refer to him now. Likewise, Buck Rogers does not yet exist; but we can refer to him now. Formally:

- For every time t, if NAME exists at that time, then f maps (NAME, t) onto that existing individual. For instance, Eric exists now; so f(NAME, t) maps "Eric" onto Eric.

- For every time t, if NAME no longer exists, then f maps (NAME, t) onto the last individual named by NAME. For example, Plato no longer exists; at every time after he exists, "Plato" refers to the last thing named by "Plato".

- For every time t, if NAME does not yet exist, then f maps (NAME, t) onto the earliest thing named by NAME. For example, Buck Rogers does not yet exist (he will indeed exist later); at every time before he exists, "Buck Rogers" refers to the earliest thing named by "Buck Rogers".

This is illustrated in Table 1.

A predicate is a noun or an adjective. The term PRED is a variable that ranges over predicates. For every PRED, the reference function f maps (PRED, time) pair onto the extension of that PRED at that time. The extension is a set of individuals in that time. The extension of the predicate "thing" is the most general extension. At any time t, f maps ("thing", t) onto δ(t). It maps "thing" onto the set of individuals at that time.

Table 1 shows a sample reference function. From the reference function, it is easy to infer the underlying universe (the underlying model).

	t1	t2	t3	t4	t5	t6	t7
"Plato"	P1	P2	P2	P2	P2	P2	P2
"Eric"	E1	E1	E1	E2	E3	E3	E3
"Buck"	B1	B1	B1	B1	B1	B1	B2
"Satan"	S1	S2	S3	S4	S5	S6	S7
"thing"	{S1, P1}	{S2, P2}	{S3, E1}	{S4, E2}	{S5, E3}	{S6, B1}	{S7, B2}
"nice"	{P1}	{P2}	{E1}	{}	{}	{B1}	{}
"mean"	{S1}	{S2}	{S3}	{S4, E2}	{S5, E3}	{S6}	{S7, B2}

Table 1. A sample temporal reference function and model.

3. Truth Conditions

3.1 Truth and Tense

We provide truth conditions for a variety of tensed sentences. These truth conditions are evaluated relative to a time t and a reference function f. The time t is the time of the utterance of the sentence (which may often be the time of its interpretation). Perhaps more generally, the time t is *now*.

We use various informal abbreviations to make the truth conditions easier to read:

(there is some x at t)(...x...)	means	(there is $x \in \delta(t)$)(...x...);
x is NAME at t	means	($x = f(\text{NAME}, t)$);
x is PRED at t	means	($x \in f(\text{PRED}, t)$).

3.2 Present Tense

Any present tense sentence has the form <NAME is PRED>. For example, "Eric is nice". Such a sentence is true at time t given reference function f iff there is a thing to which NAME refers at t and that thing is in the extension of PRED at t. Informally,

the value of <NAME is PRED> at t given f
= (there is some x at t)(x is NAME at t & x is PRED at t).

More formally,

the value of <NAME is PRED> at t given f
= (there is $x \in \delta(t)$)($x = f(\text{NAME}, t)$ & $x \in f(\text{PRED}, t)$).

3.3 Sometimes De Re

The operator "sometimes" takes us from the present time t to some time t*. Although it is usually the case that t is not t*, they might be identical. (Everything present is sometimes). For example, "Eric is sometimes nice" iff Eric exists now and he has a counterpart at some time t* and that counterpart is nice at t*. We thus write

the value of <NAME is sometimes PRED> at t given f
= (there is some x at t)(x is NAME at t &
(there is some counterpart of x at some time t*)
(that counterpart of x is PRED at t*));

the value of <NAME is sometimes PRED> at t given f
= (there is $x \in \delta(t)$)($x = f(\text{NAME}, t)$ &
(there is some time t*)(there is some $y \in \delta(t^*)$)
((y is a counterpart of x) & ($y \in f(\text{PRED}, t^*)$))).

3.4 Past Sometimes De Re

The past tense operator "was" is like "sometimes" but with the restriction of the time t* to times before the time of speech t. We thus write:

the value of <NAME was PRED> at t given *f*
= (there is some x at t)(x is NAME at t &
(there is some counterpart of x at some *past* time t*)
(that counterpart of x is PRED at t*)).

the value of <NAME was PRED> at t given *f*
= (there is $x \in \delta(t)$)($x = f$(NAME, t) &
(there is some time $t^* < t$)(there is some $y \in \delta(t^*)$)
((y is a counterpart of x) & ($y \in f$(PRED, t*)))).

Assume that the time is t4 in the model determined by Table 1. Eric exists at t4 and is mean at t4. Formally, *f* maps ("Eric", t4) onto E2, and E2 is in *f*("mean", t4). But before t4, Eric exists at t3 and is nice at t3. So E2 at t4 has a counterpart E1 at t3. And E1 is in the extension of "mean" at t3. It follows that "Eric was nice" is true at t4.

Assume that the time is t4 in the model determined by Table 1. Plato existed in the past; he does not exist at t4. He was nice in the past. However, "Plato was nice" is false at t4, because the de re reading requires Plato to exist now. Obviously, this conflicts with ordinary English. But ordinary English is not precise. When the subject of a de re past statement does not exist at the time of speech, the sentence typically gets a de dicto reading. On the de dicto reading (which we discuss below, "Plato was nice" is true.

3.5 Future Sometimes De Re

The past tense operator "will be" is like "sometimes" but with the restriction of the other times to after the time of speech. We write

the value of <NAME will be PRED> at t given *f*
= (there is some x at t)(x is NAME at t &
(there is some counterpart of x at some *future* time t*)
(that counterpart of x is PRED at t*));

the value of <NAME will be PRED> at t given *f*
= (there is $x \in \delta(t)$)($x = f$(NAME, t) &
(there is some time $t^* > t$)(there is some $y \in \delta(t^*)$)
((y is a counterpart of x) & ($y \in f$(PRED, t*)))).

Now is the time of speech. Suppose Eric exists now and will be retired after now. Then "Eric will be retired" is true now.

Now is the time of speech. Suppose Buck Rogers exists in the future (he does not exist now) and will be on Mars at some time after now. Then "Buck Rogers will be on Mars" is false now, because the de re reading requires Buck Rogers to exist now.

3.6 Always De Re

On the de re reading, a sentence of the form <NAME is always PRED> means that NAME exists now and every counterpart of NAME is PRED. The de re reading does not imply that NAME always exists. It merely implies that at every time at which NAME exists, NAME is PRED at that time. We write

the value of <NAME is always PRED> at *t* given *f*
= (there is some x at t)(x is NAME at t &
(for every counterpart of NAME, that counterpart is PRED));

the value of <NAME is always PRED> at t given *f*
= (there is x $\in$ δ(t))(x = *f*(NAME, t) &
(for every time t*)(for every y in δ(t*)),
(if y is a counterpart of x, then y is PRED at t*)).

For example, in the model implied by Table 1, Satan exists at every time and is mean at every time at which he exists; it is true at any time that "Satan is always mean".

For another example, Plato exists at only some times in Table 1. He is nice at every time at which he exists; so "Plato is always nice"is true at any time at which he exists.

3.7 Sometimes De Dicto

When "sometimes" is used de dicto, it acts on a sentence. It does not act on a predicate. The de dicto reading of "sometimes" does not require that NAME exist now. It just requires that there be some time at which <NAME is PRED> is true. Hence

the value of "It is sometimes the case that <NAME is PRED>" at t given *f*
= (there exists time t*)(<NAME is PRED> at t* given *f*).

3.8 Past Sometimes De Dicto

When "was" is used de dicto, it acts on a sentence. It says that there is some past time at which the sentence is true; the sentence is interpreted at that past time. We write

the value of "It was the case that <NAME is PRED>" at t given *f*
= (there exists some *past* time t*)(<NAME is PRED> at t* given *f*).

For example, "It was the case that Plato is nice" is true whenever it is said. For there is a time t1 at which Plato exists and Plato is nice.

Past tense statements have both strict and colloquial values. If NAME does not exist at the time t of speech, then <NAME was PRED> is read de dicto. The colloquial value of

<NAME is PRED> is the strict value of "It was the case that <NAME is PRED>". If NAME does exist at the time t of speech, then <NAME is PRED> is read de re. For examnple, Plato does not exist at t3. Hence, when said at time t3, the colloquial value of "Plato was nice" is the strict value "It was the case that Plato is wise".

3.9 Future Sometimes De Dicto

When "will be" is used de dicto, it acts on a sentence. It says that there is some future time at which the sentence is true; the sentence is interpreted at that future time. We write

the value of "It will be the case that <NAME is PRED>" at t given f
= (there exists some *future* time t*)(<NAME is PRED> at t* given f).

Future tense statements have colloquial values like past tense statements. They are read de re if the object exists now; de dicto if it does not. If it is said at time t3, "Buck Rogers will be nice" has to be read de dicto to come out true.

3.10 Always De Dicto

When "always" is used de dicto, it says that a sentence is always true. It is true at every time. For a sentence of the form "It is always the case that NAME is PRED" to be true, NAME has to exist at every time (it has to be sempiternal) and at every time at which NAME exists, it has to be PRED. It is invariably PRED. We write

the value of "It's always the case that <NAME is PRED>" at t given f
= (for every time t*)(<NAME is PRED> at t* given f).

More formally,

the value of "It's always the case that <NAME is PRED>" at t given f
= (for every time u)
(there is $x \in \delta(u)$)($x = f(\text{NAME}, u)$ & $x \in f(\text{PRED}, u)$)

For example, in the model implied by Table 1, Satan exists at every time and is mean at every time in which he exists. So, "It is always the case that Satan is mean" is true. Note that it is also true that "Satan is always mean".

For example, Plato exists only at some times; he is nice at every time at which he exists. But since he does not exist at every time, he is not nice at every time. So, "It is always the case that Plato is nice" is false. For there are times at which he does not exist, and at those times, he is not wise. But is is true that "Plato is always nice". For he is nice at every time at which he exists. Thus the de re and de dicto uses of "always" are distinct.