Sets – Exercises

Exercises for Chapter 1 of Steinhart, E. (2009) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 1)

1. Collections

Write out the following:

The set of A: **{A}**

The set of the set of A: **{{A}}**

The set of A and B: **{A, B}**

The set of both A and the set of A: **{A, {A}}**

The set of A, B, and C: **{A, B, C}**

If x is {A, B} and y is {C, D} then write out:

{x} = **{{A, B}}**

{x, y} = **{{A,B}, {C, D}}**

{{x}} = **{{{A, B}}}**

{{x}, y} = **{{{A,B}}, {C, D}}**

Answer the following (true or false):

1 = {1}?	**False**	{1} = {{1}}?	**False**
{1, 1} = {1, {1}}?	**False**	{1, B, 2} = {2, 1, B}?	**True**
{A, A} = {A}?	**True**	{A, A} = {{A}}?	**False**

2. Membership

True or false:

Is A ∈ {A}?	**True**	Is {A} ∈ {{A}}?	**True**
Is A ∈ {{A}}?	**False**	Is {B} ∈ {{A}, {B}}?	**True**
Is {A, B} ∈ {A, B}?	**False**	Is {} ∈ {A}?	**False**

3. Set Builders

Using the set Y = {1, A, 2, B, 3, C}, write out the following sets:

{ x ∈ Y | x is a letter} = **{A, B, C}**

$\{ x \in Y \mid x \text{ is a number}\}$ = **{1, 2, 3}**

If X = {A, B, C, 1, 2, 3}, Y = {A, B, C}, and Z = {1, 2, 3}, then write:

$\{x \in X \mid x \text{ is also in } Y\}$ = **{A, B, C}**

$\{x \in X \mid x \in Z\}$ = **{1, 2, 3}**

4. Unions

Write out:

$\{a, b\} \cup \{1\}$ = **{a, b, 1}**

$\{a, b\} \cup \{c, d\}$ = **{a, b, c, d}**

$\{a\} \cup \{b\} \cup \{c\}$ = **{a, b, c}**

$\{\{A\}\} \cup \{\{B\}\}$ = **{{A}, {B}}**

$\{a\} \cup \{\{1\}\}$ = **{a, {1}}**

$\{A\} \cup \{\{A\}\}$ = **{A, {A}}**

5. Intersections

$\{a, b\} \cap \{a\}$ = **{a}**

$\{a, b\} \cap \{\{a\}\}$ = **{}**

$\{1, 2, 3\} \cap \{a, b\}$ = **{}**

$\{\{\}, A\} \cap \{A, \{\}\}$ = **{{}, A}**

$\{\{\}, 2, 3\} \cap \{\{\}\}$ = **{{}}**

$\{a, b, c\} \cap \{b, c, d\}$ = **{b, c}**

6. Subsets

True or false.

{A, B} is a subset of {A, B, C}? **True**

{A} is a subset of {A, B}? **True**

A is a subset of {A}? **False**

{A} is a subset of {{A}}? **False**

{A} is a subset of {A, {A}}? **True**

{A, B} is a subset of {A, B}? **True**

Write the subsets of {1, 2}: **{}, {1}, {2}, {1, 2}**

7. Rank

Assume that A, B, and C are individuals on rank 0.

rank of {A} = **1**

rank of {{{B}}, {C}} = **3**

rank of {A, {A}} = **2**

rank of {{A, B}, {C}} = **2**

8. Power Sets

The power set of {1, A} is: **{{}, {1}, {A}, {1, A}}**

The power set of {Q} is: **{{}, {Q}}**

The power set of {{}} is: **{{}, {{}}}**

Write the power set of the power set of {A}:

the power set of {A} = {{}, {A}}
the power set of {{}, {A}} = **{{}, {{}}, {{A}}, {{}, {A}}}**

9. Some Transformations of Sets

Suppose X = {A, B, C}.
Write the set that results from replacing each $x \in$ X with {x}:

{{A}, {B}, {C}}

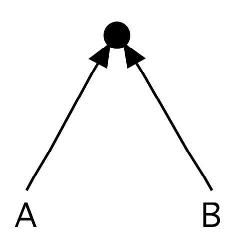
Suppose X = {{{A}, {B}}, {{C}}}.
Write the set that results from replacing each $x \in$ X with $\cup x$:

{{A, B}, {C}}

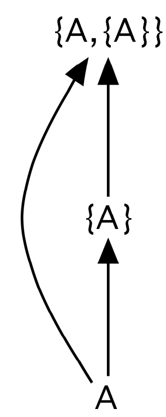
10. Diagramming Sets

Use names or dots for sets and an arrow from x to y iff x is a member of y.

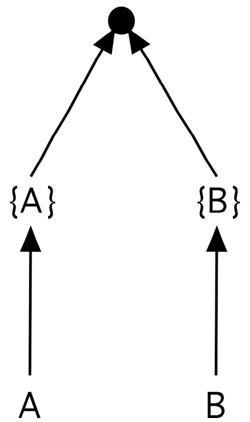
Draw the diagram for {A, B}.



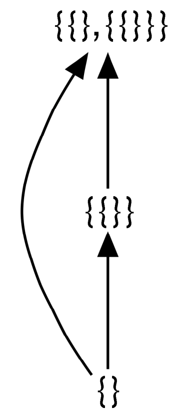
Draw the diagram for {A, {A}}



Draw the diagram for {{A}, {B}}.



Draw the diagram for {{}, {{}}}.



11. Sets and Selections

Fill in the table with 0s and 1s to express all selections. Write the selected set in the rightmost cell of each row.

{}	A	{A}	
1	1	1	{{}, A, {A}}
1	1	0	{{}, A}
1	0	1	{{}, {A}}
1	0	0	{{}}
0	1	1	{A, {A}}
0	1	0	{A}
0	0	1	{{A}}
0	0	0	{}

Now write the set of all sets from the rightmost cell of each row:

{{}, {{A}}, {A}, {A, {A}}, {{}}, {{}, {A}}, {{}, A}, {{}, A, {A}}}

12. Numbers as Sets

Using the idea that *n* is the set of all numbers less than *n*, write out:

0 = **{}**

1 = **{0}**

2 = **{0, 1}**

3 = **{0, 1, 2}**

4 = **{0, 1, 2, 3}**

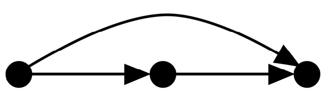
13. Diagramming Numbers as Sets

Draw a diagram for each of the numbers in exercise 12 above:

0 

1 

2



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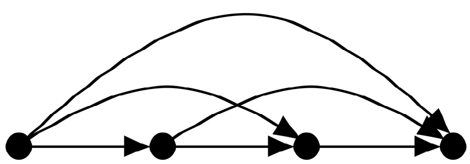

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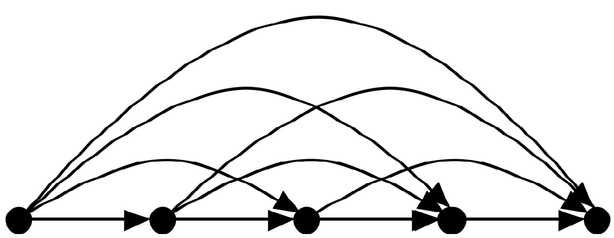


14. Iteration versus Accumulation

An *iterative hierarchy* says that every next level is just the power set of the previous level.

Consider this iterative hierarchy:

H(0) = {A};   H(n+1) = pow H(n).

Write out levels H(0), H(1), and H(2) of this iterative hierarchy:

H(0) = **{A}**

H(1) = **{{}, {A}}**

H(2) = **{{}, {{}}, {{A}}, {{}, {A}}}**

A *cumulative hierarchy* says that every next level is the power set of the previous level unioned with the previous level.

Consider this cumulative hierarchy:

K(0) = {A};   K(n+1) = pow K(n) $\cup$ K(n).

Write out levels K(0), K(1), and K(2) of this cumulative hierarchy:

K(0) = {A}

K(1) = {{}, A, {A}}

Write out pow K(1). (How can exercise 11 help you?)

{{}, {{A}}, {A}, {A, {A}}, {{}}, {{}, {A}}, {{}, A}, {{}, A, {A}}}

K(2) = **{{}, A, {{A}}, {A}, {A, {A}}, {{}}, {{}, {A}}, {{}, A}, {{}, A, {A}}}**

Give an example of an object that appears on K(1) but not on H(1): **A**

List all objects that appear on K(2) but not on H(2):

A, {A}, {A, {A}}, {{}, A}, {{}, A, {A}}

Explain why K(n+1) is richer than H(n+1) for $n > 0$:

Because both members and subsets of K(n) appear on K(n+1), while only subsets of H(n) appear on H(n+1).

15. Ordered Pairs

Diagram (Sue, Bob)

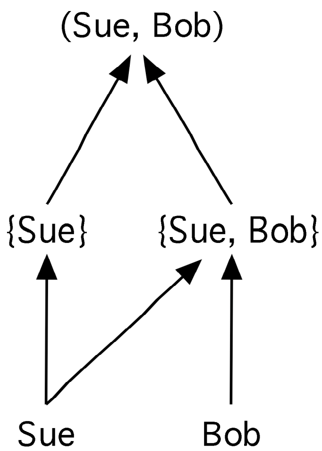


Diagram (Bob, Bob)

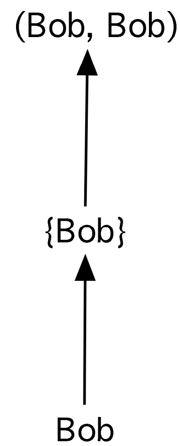


Diagram (Sue, {Sue})

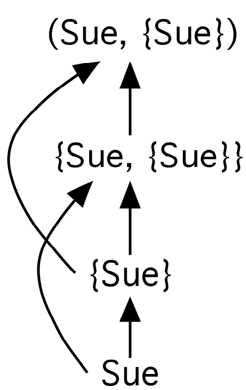
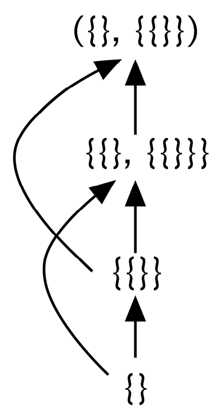


Diagram ({}, {{}})



16. Cartesian Products

Write the Cartesian Product $\{A, B\} \times \{1, 2\}$.

{(A, 1), (A, 2), (B, 1), (B, 2)}

Write the Cartesian Product $\{\text{Abe}, \text{Bob}, \text{Sue}\} \times \{\text{Happy}, \text{Sad}\}$.

{ (Abe, Happy), (Abe, Sad),
(Bob, Happy), (Bob, Sad),
(Sue, Happy), (Sue, Sad)}